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The Dial-a-Ride Problem with Meeting Points: A problem formulation for Shared Demand Responsive Transit

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Research Contribution

The contribution of this research is the extension of the DARP in a way that it includes **meeting points**. This means that the optimal meeting point for a request is found while solving the DARPmp, instead of first defining a meeting point for certain requests and then solving the original DARP. The contribution consists of the formulation of a **mixed-integer non-linear program** for DARPmp and the linearization of this problem formulation to get a **mixed-integer linear program**. It also consists of the formulation of a number of preprocessing steps and valid inequalities for the DARPmp. Next to that, two **meta-heuristic algorithms** are proposed to solve the DARPmp for large instances.

Problem Formulation

The three-index formulation of Cordeau (12) is extended as follows:

$$\min \sum_{k\in K}\sum_{(i,j)\in\mathcal{A}} c_{ij}^k x_{ij}^k$$

$$\begin{aligned}
\text{s.t.}\quad & \sum_{k\in K}\sum_{m\in\mathcal{N}} y_{im}^k = 1, && \forall i\in P\cup D\\
& \sum_{j:(j,m)\in\mathcal{A}} x_{jm}^k \geq y_{im}^k && \forall i\in V, m\in\mathcal{N}, k\in K\\
& \sum_{j\in P\cup\mathcal{M}_p} x_{0j}^k = \sum_{i\in D\cup\mathcal{M}_d} x_{i,2n+1}^k = 1 && \forall k\in K\\
& \sum_{j:(j,i)\in\mathcal{A}} x_{ji}^k - \sum_{j:(i,j)\in\mathcal{A}} x_{ij}^k = 0 && \forall i\in P\cup D\cup\mathcal{M}, k\in K\\
& \sum_{m\in\mathcal{N}} y_{im}^k - \sum_{m\in\mathcal{N}} y_{n+i,m}^k = 0 && \forall i\in P, k\in K\\
& \sum_{m\in\mathcal{N}}\sum_{i\in V} y_{im}^k \geq 1 && \forall k\in K\\
& y_{im}^k + y_{i+n,m}^k \leq 1 && \forall i\in P, k\in K, m\in\mathcal{N}\\
& s_{im} = \sum_{k\in K} y_{im}^k d_i && \forall i\in P\cup D, m\in\mathcal{N}\\
& z_{im} = \sum_{k\in K} y_{im}^k q_i && \forall i\in P\cup D, m\in\mathcal{N}\\
& u_j^k \geq (u_m^k + \sum_{i\in V} s_{im} + t_{mj}) x_{mj}^k && \forall (m,j)\in\mathcal{A}, k\in K\\
& w_j^k \geq (w_m^k + \sum_{i\in V} z_{ij}) x_{mj}^k && \forall (m,j)\in\mathcal{A}, k\in K\\
& w_0^k = w_{2n+1}^k = 0 && \forall k\in K\\
& u_{2n+1}^k - u_0^k \leq T_k && \forall k\in K\\
& (e_i + p_{im}) y_{im}^k \leq u_m^k \leq l_i + (1 - y_{im}^k) T_k && \forall i\in P, m\in\mathcal{N}, k\in K\\
& e_i y_{im}^k \leq u_m^k \leq l_i - p_{im} + (1 - y_{im}^k) T_k && \forall i\in D, m\in\mathcal{N}, k\in K\\
& \sum_{m\in\mathcal{N}} y_{i+n,m}^k u_m^k - \sum_{m\in\mathcal{N}} (y_{i,m}^k (u_m^k + \sum_{i\in V} s_{im})) \leq L && \forall i\in P, k\in K\\
& t_{m,\mu} \leq \sum_{m\in\mathcal{N}} y_{i+n,m}^k u_m^k - \sum_{m\in\mathcal{N}} (y_{i,m}^k (u_m^k + \sum_{i\in V} s_{im})) && \forall i\in P, k\in K\\
& \max\{0, \sum_{i\in V} z_{im}\} \leq w_m^k \leq \min\{Q_k, Q_k + \sum_{i\in V} z_{im}\} && \forall m\in\mathcal{N}, k\in K\\
& c_{im} y_{im}^k \leq C^{max} && \forall i\in V, m\in\mathcal{N}, k\in K\\
& c_{im} y_{im}^k \geq y_{im}^k C^{min} && \forall i\in V, m\in\mathcal{N}, i\neq m, k\in K\\
& v_i^k = \sum_{m\in\mathcal{N}} y_{im}^k p_{im} + \sum_{\mu\in\mathcal{N}} y_{i+n,\mu} p_{i+n,\mu} && \forall i\in P, k\in K\\
& v_i^k \leq W && \forall i\in P, k\in K\\
& x_{ij}^k \in \{0,1\} && \forall (i,j)\in\mathcal{A}, k\in K\\
& y_{im}^k \in \{0,1\} && \forall i\in V, m\in\mathcal{N}, k\in K
\end{aligned}$$

Toy Network

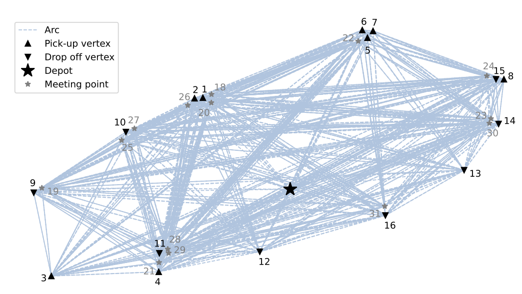

Figure 1: Toy network to validate the formulation

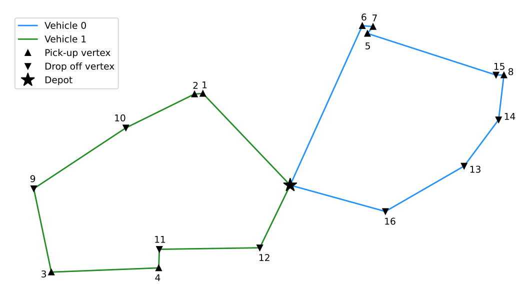

Figure 2: DARP solution for the toy network

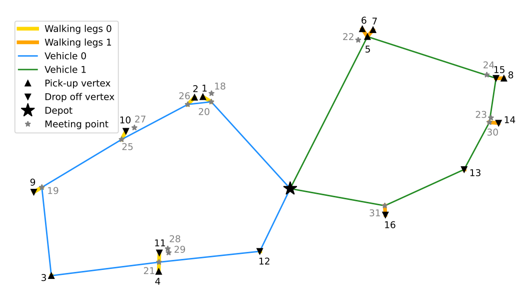

Figure 3: DARPmp solution for the toy network

The routes of the DARPmp result in a total travel cost of 96.14, which is considerably lower that the total travel cost 101.46 of the original DARP. It can be observed that several meeting points are used, but not all of them.

Experiments in Larger Networks

Table 1: Computational results of the Branch-and-cut (B&C) algorithm without valid inequalities, the Branch-and-cut algorithm with valid inequalities, Tabu Search algorithm 1 and Tabu Search algorithm 2. OFV=Objective Function Value.

	B&C		B&C PV		TS1				TS2			
Instance	**OFV**	**Time (s)**	**OFV**	**Time (s)**	**OFV**	**Gap**	**AVG**	**Time (s)**	**OFV**	**Gap**	**AVG**	**Time (s)**
a2-16-24	-	-	285,53	30,77	286,57	0,361%	286,57	31,71	286,57	0,361%	288,41	160,66
a2-20-30	-	-	337,62	74,53	337,62	0,000%	337,62	139,09	337,62	0,000%	337,62	1050,51
a2-24-36	-	-	423,15	134,65	423,53	0,090%	425,29	157,14	423,20	0,011%	424,88	918,83
a3-18-27	-	-	295,20	2414,48	295,21	0,003%	299,53	22,42	301,81	2,240%	302,70	77,40
a3-24-36	-	-	336,57	14036,27	342,12	1,650%	342,99	154,63	341,13	1,354%	343,83	692,67
a3-30-45	-	-	-	-	486,13	-	486,94	210,40	486,42	-	491,90	1112,18
a3-36-54	-	-	-	-	583,38	-	584,52	328,55	556,94	-	559,73	3226,27
a4-16-24	-	-	-	-	276,32	-	279,44	14,07	276,32	-	279,46	38,20
a4-24-36	-	-	-	-	369,18	-	378,66	66,13	375,61	-	379,83	251,16
a4-32-48	-	-	-	-	478,02	-	486,93	138,21	475,46	-	494,11	740,74
a4-40-60	-	-	-	-	561,28	-	565,22	421,38	562,30	-	568,96	2239,43
a4-48-72	-	-	-	-	675,10	-	685,48	850,51	676,42	-	680,51	9051,69

Key Findings

- Preprocessing steps and valid inequalities can be used to improve the computational performance of the DARPmp when solved with Branch and Cut.
- For larger instances, two Tabu Search frameworks were proposed to approximate the optimal solution for the DARPmp.
- The first algorithm finds solutions with an optimality gap ranging from 0% to 1.65%. The run times for this algorithm range from 14.07 seconds to 850.51 seconds.
- The second algorithm finds solutions with an optimality gap ranging from 0% to 2.24% and has run times ranging from 38.20 to 9051.69 seconds.

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